PROCESSAMENTO DIGITAL DE SINAL

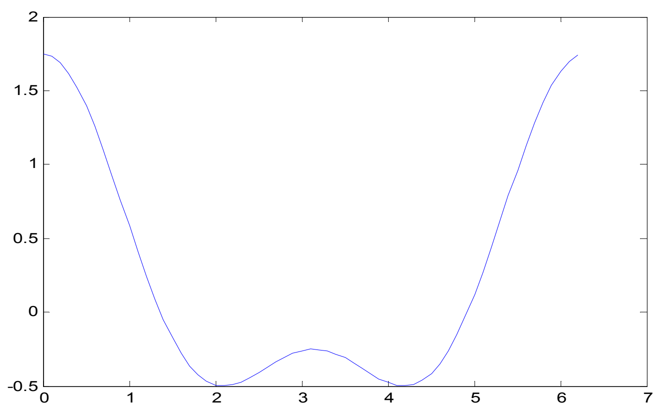
Exame de 2002/01/24 – Resolução

1.

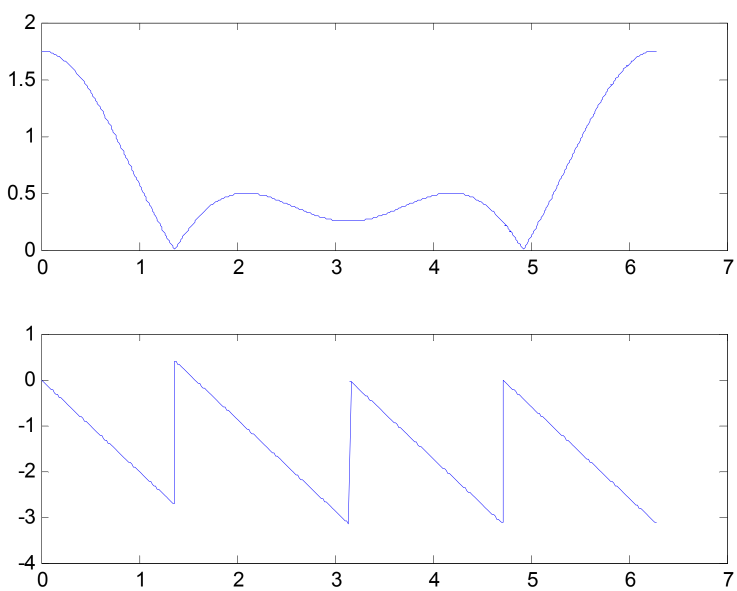
a. $h(n)=\dfrac{\delta(n)+2\cdot\delta(n-1)+\delta(n-2)+2\cdot\delta(n-3)+\delta(n-4)}{4}$.

b.
$$H(e^{j\omega})=\frac{1+2\cdot e^{-j\omega}+e^{-j2\omega}+2\cdot e^{-j3\omega}+e^{-j4\omega}}{4}=\frac{\left(e^{j2\omega}+2\cdot e^{j\omega}+1+2\cdot e^{-j\omega}+e^{-j2\omega}\right)}{4}e^{-j2\omega}$$
$$H(e^{j\omega})=\frac{1+4\cos(\omega)+2\cos(2\omega)}{4}e^{-j2\omega}$$

Graficamente, vem:



Separando o módulo e a fase, virá:



c. $H(z) = \dfrac{1 + 2z^{-1} + z^{-2} + 2z^{-3} + z^{-4}}{4}$

$X(z) = \dfrac{1 - 2z^{-1} + 3z^{-2}}{2}$

Como Y(z) = X(z)·H(z), vem

$Y(z) = \dfrac{1 + 6z^{-3} + 4z^{-5} + 3z^{-6}}{8}$

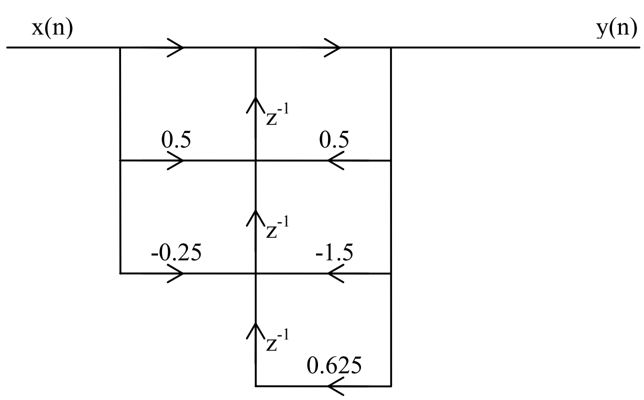
donde se tira

$y(n) = \dfrac{\delta(n) + 6\delta(n-3) + 4\delta(n-5) + 3\delta(n-6)}{8}$.

2. $y(n) = x(n) + \frac{1}{2}x(n-1) - \frac{1}{4}x(n-2) + \frac{1}{2}y(n-1) - \frac{3}{2}y(n-2) + \frac{5}{8}y(n-3)$

a. $H(z) = \dfrac{1 + 0.5z^{-1} - 0.25z^{-2}}{1 - 0.5z^{-1} + 1.5z^{-2} - 0.625z^{-3}}$

b.



3.

a. $H(z) = z^{-5}(z - 0.1962 - j0.2863)(z - 0.1962 + j0.2863)(z - 1.6288 - j2.3766)(z - 1.6288 + j2.3766)(z + 1)$

$H(z) = z^{-5}(z^2 - 0.3924z + 0.12046)(z^2 - 3.2576z + 8.3012)(z + 1)$

Agora há 3 hipóteses:

i. $H(z) = z^{-5}(z^4 - 3.65z^3 + 9.7z^2 - 3.65z + 1)(z + 1)$

ii. $H(z) = z^{-5}(z^3 + 0.6076z^2 - 0.2719z + 0.1204)(z^2 - 3.2576z + 8.3012)$

iii. $H(z) = z^{-5}(z^3 - 2.2576z^2 + 5.0436z + 8.3012)(z^2 - 0.3924z + 0.12046))$

Das 3 hipóteses, chega-se ao resultado final:

$$H(z) = z^{-5}(z^5 - 2.65z^4 + 6.05z^3 + 6.05z^2 - 2.65z + 1)$$

Donde se tira,

$$H(z) = (1 - 2.65z + 6.05z^2 + 6.05z^3 - 2.65z^4 + z^5)$$

e finalmente

$$h(n) = \delta(n) - 2.65\delta(n-1) + 6.05\delta(n-2) + 6.05\delta(n-3) - 2.65\delta(n-4) + \delta(n-5)$$

b. Como a resposta impulsional é simétrica, então o sistema é de fase linear.

c. É possível decompor este sistema em dois sistemas, cada um deles com fase linear; basta ver a primeira hipótese considerada na alínea a) desta questão.

4.

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k) W_N^{-nk}$$

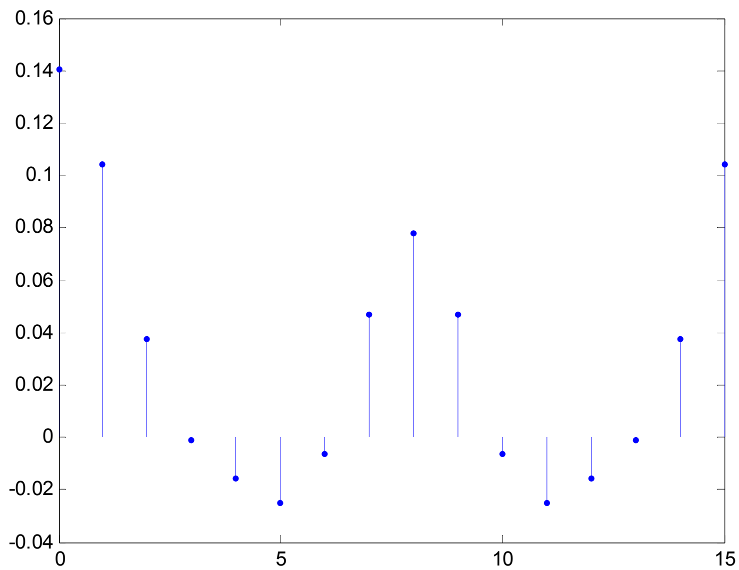
$$x(n) = \frac{1}{16}\left(0.5 + 0.25W_{16}^{-n} + 0.5W_{16}^{-2n} + 0.125W_{16}^{-4n} + 0.125W_{16}^{-12n} + 0.5W_{16}^{-14n} + 0.25W_{16}^{-15n}\right)$$

a. $$x(n) = \frac{1}{16}\left(0.5 + 0.25W_{16}^{-n} + 0.5W_{16}^{-2n} + 0.125W_{16}^{-4n} + 0.125W_{16}^{4n} + 0.5W_{16}^{2n} + 0.25W_{16}^{n}\right)$$

$$x(n) = \frac{1}{16}\left(0.5 + 0.25e^{j\frac{2\pi}{16}n} + 0.5e^{j\frac{2\pi}{16}2n} + 0.125e^{j\frac{2\pi}{16}4n} + 0.125e^{-j\frac{2\pi}{16}4n} + 0.5e^{-j\frac{2\pi}{16}2n} + 0.25e^{-j\frac{2\pi}{16}n}\right)$$

$$x(n) = \frac{1}{16}\left(0.5 + 0.5\cos\left(\frac{\pi}{8}n\right) + \cos\left(\frac{\pi}{4}n\right) + 0.25\cos\left(\frac{\pi}{2}n\right)\right)$$

b.



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5.

a. $H_a(s) = \dfrac{1}{1+0.125s} = \dfrac{8}{s+8} \quad \Rightarrow \Omega_c = 8\ \text{rads}^{-1} \quad \therefore \quad F_c = 1.273\ \text{Hz}$

b. $H(z) = \dfrac{1.6}{1 - e^{-1.6} z^{-1}}$.