Numeration Systems and Binary Codes

numbering system that allowed, through ten distinct symbols (digits), represent a specific quantity based on another taken as unity.

Decimal System - Allows represent any amount through a weighted sum of base 10 powers.

$$852=8\times10^{2}+5\times10^{1}+2\times10^{0}$$

$$0.852=8\times10^{-1}+5\times10^{-2}+2\times10^{-3}$$

Characteristics of a decimal number:

- Coefficients of base 10 powers whose exponents grow with steps of one unit from right to left.

- Fractional part are coefficients of base 10 powers whose exponents decrease in increments of one unit from left to right

Other numeration bases can be used: base 2, base 8 and base 16.

The numeration binary system includes only two distinc simbols: the zero (0) and the one (1)

digit -> *bit*

The contribution of a bit in a binary number depends on the relative position it occupies.

Decimal equivalent: $10011_2=1x2^4+0x2^3+0x2^2+1x2^1+1x2^0=19_{10}$

Most significant bit (MSB) vs. Less significant bit (LSB)

Binary comma concept:

$0,10011_2=1x2^{-1}+0x2^{-2}+0x2-^{3}+1x2^{-4}+1x2^{-5}=0.59375_{10}$

What is the biggest decimal number that one can write with only two digits?

In a binary number which the largest integer decimal number you can represent with n bits?

Depending on the number of bits of a word on base 2: Nibble or Byte

1kb = ?   1Mb= ?

Decimal	**Binary**	**Decimal**	**Binary**
0	0	9	1001
1	1	10	1010
2	10	11	1011
3	11	12	1100
4	100	13	1101
5	101	14	1110
6	110	15	1111
7	111	16	10000
8	1000	17	10001

OCTAL and HEXADECIMAL - are commonly used as an alternative representation of binary numbers.

Decimal	Binary	Octal	Hexad.	Decimal	Binary	Octal	Hexad.
0	00000	0	0	16	10000	20	10
1	00001	1	1	17	10001	21	11
2	00010	2	2	18	10010	22	12
3	00011	3	3	19	10011	23	13
4	00100	4	4	20	10100	24	14
5	00101	5	5	21	10101	25	15
6	00110	6	6	22	10110	26	16
7	00111	7	7	23	10111	27	17
8	01000	10	8	24	11000	30	18
9	01001	11	9	25	11001	31	19
10	01010	12	A	26	11010	32	1A
11	01011	13	B	27	11011	33	1B
12	01100	14	C	28	11100	34	1C
13	01101	15	D	29	11101	35	1D
14	01110	16	E	30	11110	36	1E
15	01111	17	F	31	11111	37	1F

$27,63_8=2x8^1+7x8^0+6x8^{-1}+3x8^{-2}=23.796875_{10}$

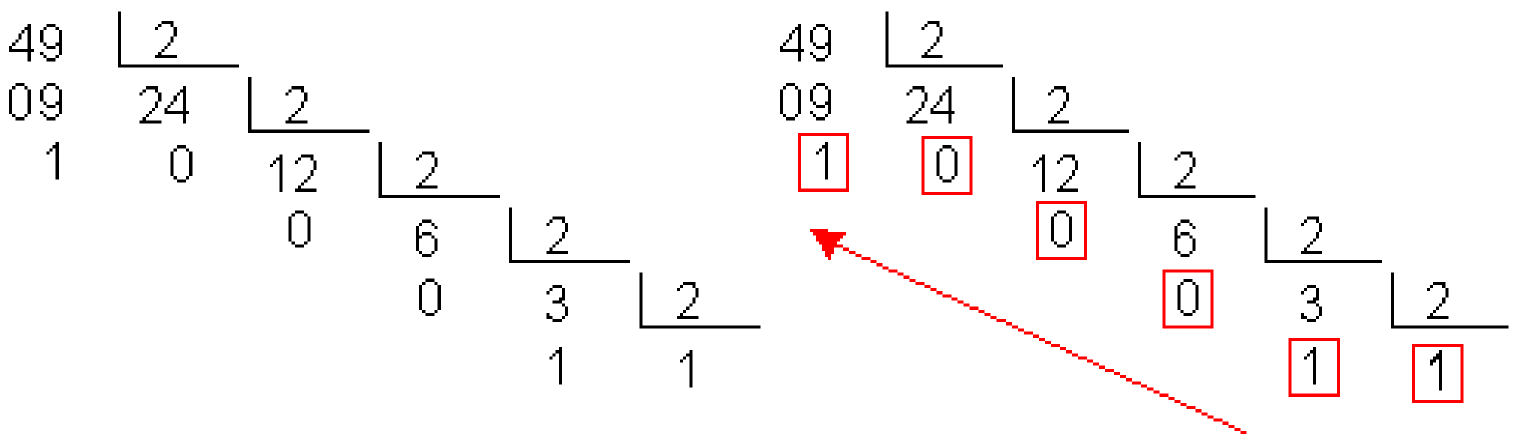
$9CA,3B_{16}=9x16^2+12x16^1+10x16^0+3x16^{-1}+11x16^{-2}=2506.23046875_{10}$

Convertion Techniques between Bases:

Binary < > Decimal

Binary - > Decimal : Weighted Sum

Decimal -> Binary: Division (Multiplication)



If the number to convert from base 10 to base 2 is not pure integer but has a fractional part, the conversion is done in two stages

Integer Part + Decimal Part

$0{,}703125 \times 2 = 1{,}40625$	$0{,}703125 \times 2 = \boxed{1}{,}40625$
$0{,}40625 \times 2 = 0{,}8125$	$0{,}40625 \times 2 = \boxed{0}{,}8125$
$0{,}8125 \times 2 = 1{,}625$	$0{,}8125 \times 2 = \boxed{1}{,}625$
$0{,}625 \times 2 = 1{,}25$	$0{,}625 \times 2 = \boxed{1}{,}25$
$0{,}25 \times 2 = 0{,}5$	$0{,}25 \times 2 = \boxed{0}{,}5$
$0{,}5 \times 2 = 1{,}0$	$0{,}5 \times 2 = \boxed{1}{,}0$

Decimal -> Octal ou Hexadecimal $2655{,}6396484375_{10}$

2655 | 16
$\boxed{15}$ (F) 165 | 16
$\boxed{5}$ $\boxed{10}$ (A)

$0{,}6396484375 \times 16 = \boxed{10}{,}234375$

$0.234375 \times 16 = \boxed{3}{,}75$

$0{,}75 \times 16 = \boxed{12}{,}0$

76 | 8
$\boxed{4}$ 9 | 8
$\boxed{1}$ $\boxed{1}$

$0{,}21875 \times 8 = \boxed{1}{,}75$

$0{,}75 \times 8 = \boxed{6}{,}0$

Binary <-> Hexadecimal

100100110,101

0001 0010 0110 , 1010

1 2 6 , A

Form groups of 4 bits.

If necessary add zeros to MSB's left or to LSB's right of the fractional part

Convert each group in the hexadecimal equivalent value.

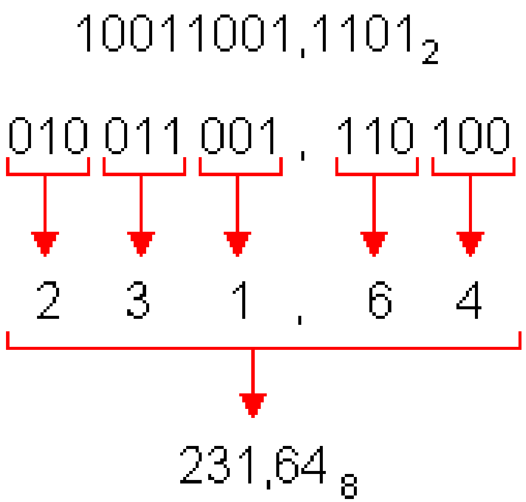
23C,F4

0010 0011 1100 , 1111 0100

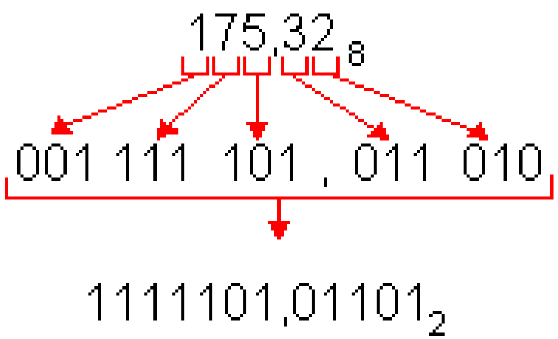
$1000111100,111101_2$

Binary <-> Octal

Binary → Octal



Octal → Binary



Negative Numbers Representation in Base 2

Signal and Magnitude

0 1 1 0 1 1 0 0 1 = + 217

1 1 1 0 1 1 0 0 1 = -217

Complement to 1

0 1 1 0 1 1 0 0 1 = + 217

Complement from 1

1 0 0 1 0 0 1 1 0 = -217

Complement to 2

0 1 1 0 1 1 0 0 1 = + 217

Complement from 2

1 0 0 1 0 0 1 1 1 = -217

The representation of negative numbers in base 2 only has meaning if the number of bits used in the codification is set !

1101110 -> 110 (8 bit) ou -18 (7 bit)

Binary Codes

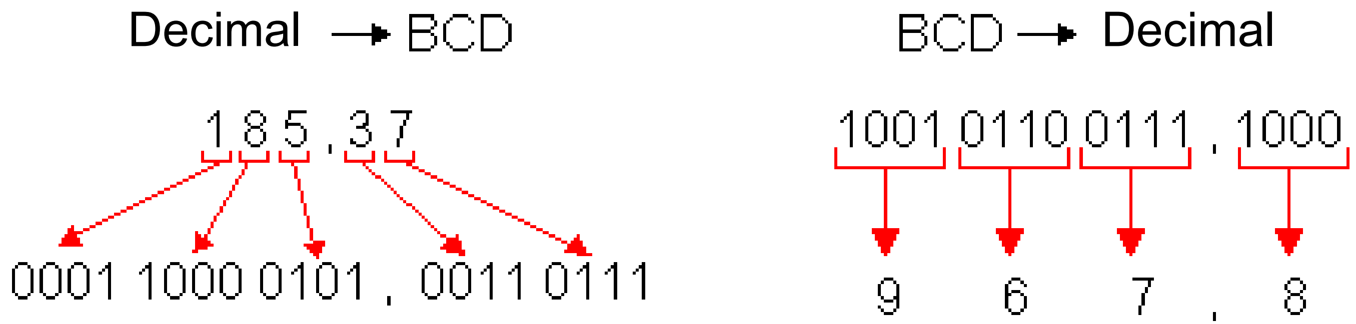
set of symbols (alphabet) and rules that allow sorting and combining these symbols.

The purpose of these codes is to facilitate communication between Man and machine.

Weighted

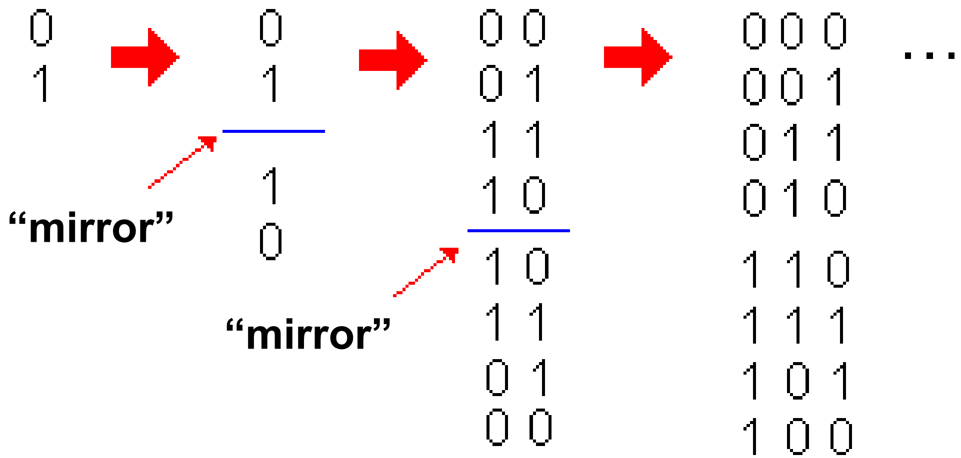
It is possible to assign weights to each of the word bits...

BCD8421 BCD 4221 BCD 5421



Non-Weighted

Gray code: it has the particularity that, between adjacent values, just change a bit.



Decimal (for reference)	Gray abcd
0	0000
1	0001
2	0011
3	0010
4	0110
5	0111
6	0101
7	0100
8	1100
9	1101
10	1111
11	1110
12	1010
13	1011
14	1001
15	1000

Another non-weighted binary code extremely widespread is the ASCII code (American Standard Code for Information Exchange)

BOOLE ALGEBRA

- Investigation of the fundamental laws of the human mind operations linked to reasoning.

- The traditional algebra operates with quantitative relationships while the Boole algebra operates with logical relations

- In Boolean algebra, functions are binary of binary variables, i.e., can only have two distinct states: True or False.

‘1’ and ‘0’ represent physical states of matter

•Besides the algebraic form, the Boolean functions may be characterized by truth table.

LOGIC OPERATORS

- In Boole algebra there are four elementary logical operators. They are the Equality, the Negation, the Union and the Intersection

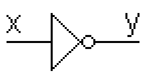
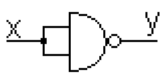
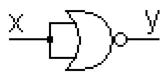
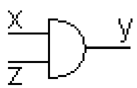
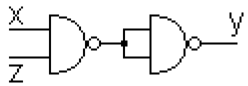
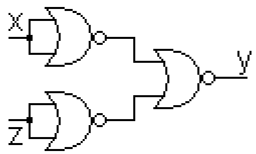
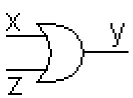
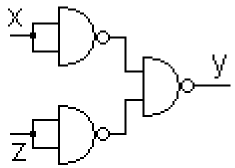
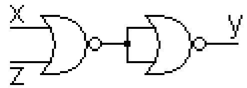
Let x and y be Boolean variables, and F (x, y) be a boolean variable Boolean function:

- Equality Operator: Truth T. + logical symbol
- Negation Operator: Truth T. + logical symbol
- Intersection Operator: Truth T. + logical symbol
- Union Operator: Truths T. + logical symbol

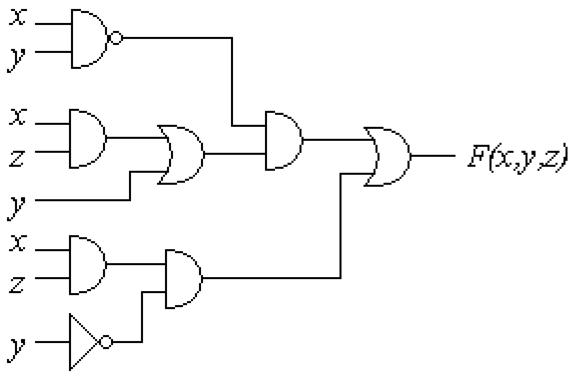
Other Logic Gates:

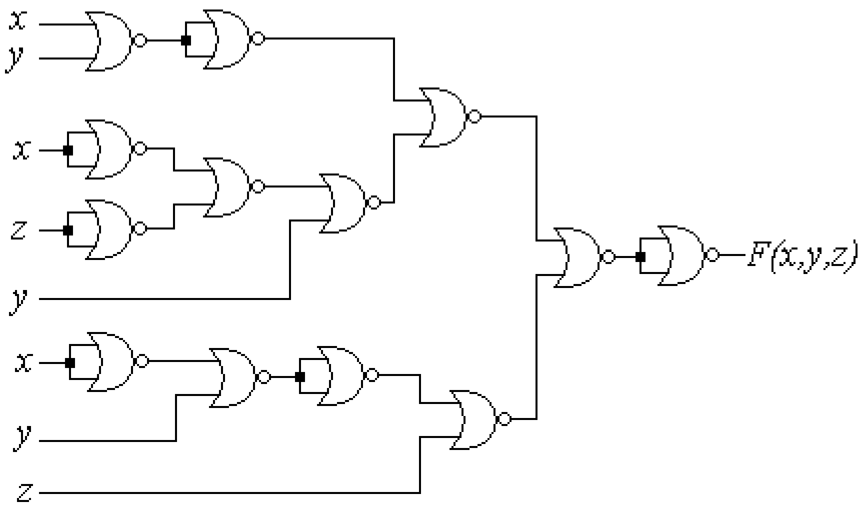
- Exclusive Or (XOR)
- Union Complement (NOR)
- Intersection Complement (NAND)

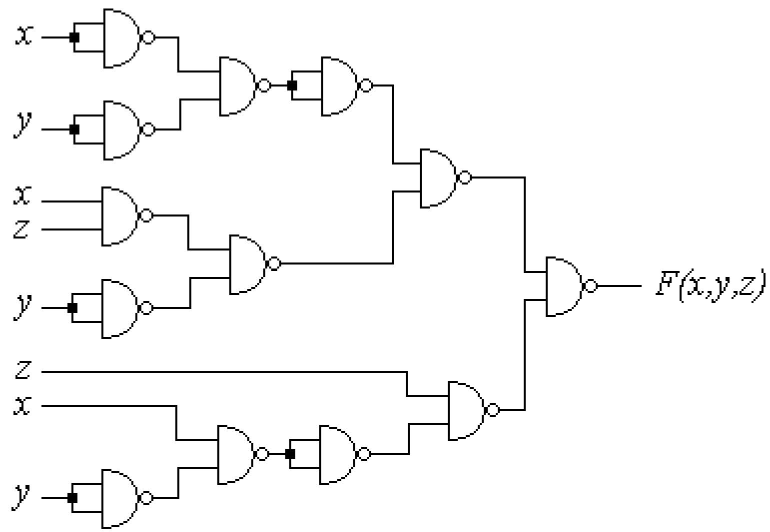
NAND and NOR gates as universal functions:

Function	NAND gates	NOR gates
		
		
		

Example







Canonical Form of a Logical Expression

• Canonical form of a Boolean function: product of sums, and sums of products on which appears all variables in each one of the terms, in its direct or complemented shape.

MINTERMS or canonical disjunctive: Sum of all logical products that give the function the value '1'.

MAXTERMS or canonical conjunctive: Multiplication of all logical sums that give the function the value '0'.

Disjunctive form

x	y	$F(x)$
0	0	0
0	1	1
1	0	1
1	1	0

$$F(x,y) = \bar{x}.y + x.\bar{y}$$

Conjunctive form

x	y	$F(x)$
0	0	0
0	1	1
1	0	1
1	1	0

$$F(x,y) = (x+y) \cdot (\bar{x}+\bar{y})$$

Identities and rules of Boolean Algebra

Postulados	
$A+0=A$	$A\cdot 0=0$
$A+A=A$	$A\cdot A=A$
$A+1=1$	$A\cdot 1=A$
$A+\overline{A}=1$	$A\cdot \overline{A}=0$

Rules	
Commutative Property	$A+B=B+A$
	$A\cdot B=B\cdot A$
Associative Property	$(A+B)+C=A+(B+C)$
	$(A\cdot B)\cdot C=A\cdot(B\cdot C)$
Distributive Property	$A\cdot(B+C)=A\cdot B+A\cdot C$
	$A+B\cdot C=(A+B)\cdot(A+C)$
Absorption	$A+A\cdot B=A$
	$A\cdot(A+B)=A$
(Some Useful theorems)	$A+\overline{A}\cdot B=A+B$
	$A\cdot(\overline{A}+B)=A\cdot B$
	$A\cdot B+\overline{A}\cdot C+B\cdot C=A\cdot B+\overline{A}\cdot C$
	$(A+B)\cdot(\overline{A}+C)\cdot(B+C)=(A+B)\cdot(\overline{A}+C)$
Leis de De Morgan	$\overline{A+B}=\overline{A}\cdot\overline{B}$
	$\overline{A\cdot B}=\overline{A}+\overline{B}$

Portas NAND

$$F(x,y,z)=\overline{(x+y)}\cdot(x\cdot z+y)+x\cdot\overline{y}\cdot z$$

$$=\overline{\overline{\overline{(x+y)}\cdot(x\cdot z+y)+x\cdot\overline{y}\cdot z}}$$

$$=\overline{\overline{\overline{(x+y)}\cdot(xz+y)}\cdot\overline{x\cdot\overline{y}\cdot z}}$$

$$=\overline{\overline{\overline{\overline{\overline{(x+y)}}}\cdot\overline{\overline{(xz+y)}}}\cdot\overline{x\cdot\overline{y}\cdot z}}$$

$$=\overline{\overline{\overline{\overline{(\overline{x}\cdot\overline{y})}}\cdot\overline{(\overline{\overline{x\cdot z}}\cdot\overline{y})}}\cdot\overline{\overline{\overline{x\cdot\overline{y}}}\cdot z}}$$

Portas NOR

$$F(x,y,z)=\overline{(x+y)}\cdot(x\cdot z+y)+x\cdot\overline{y}\cdot z$$

$$=\overline{\overline{\overline{\overline{\overline{(x+y)}\cdot(x\cdot z+y)}}+\overline{\overline{(x\cdot\overline{y}\cdot z)}}}}$$

$$=\overline{\overline{\overline{\overline{\overline{(x+y)}}+\overline{\left(\overline{\overline{xz}}+y\right)}}+\overline{\left(\overline{x\overline{y}}+z\right)}}}$$

$$=\overline{\overline{\overline{\overline{\overline{(x+y)}}+\overline{(\overline{\overline{x}+\overline{z}}+y)}}+\overline{\left(\overline{\overline{\overline{x\overline{y}}}}+z\right)}}}$$

$$=\overline{\overline{\overline{\overline{\overline{\overline{(x+y)}}}+\overline{(\overline{\overline{x}+\overline{z}}+y)}}+\overline{\left(\overline{\overline{\overline{x}+y}}+z\right)}}}$$

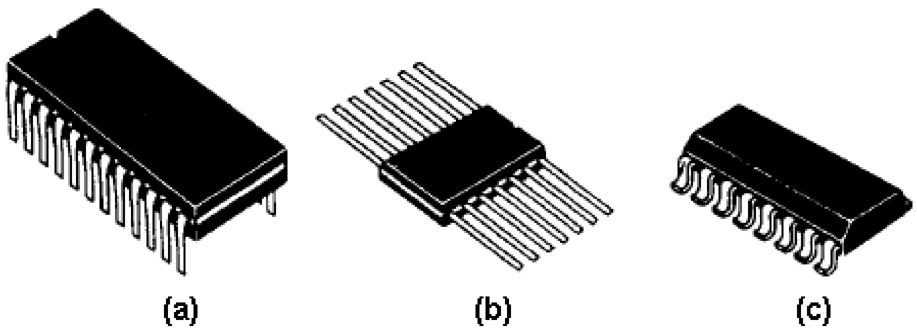
COMBINATORIAL CIRCUITS

- So far were reviewed the cornerstones that support the analysis and design of logical systems.
- From Boole algebra to digital computers!
- The state of the art of electronic digital circuits is based on a digital electronic device referred to as integrated circuit (IC).
- An integrated circuit is a complete electronic circuit made on a pastille of semiconductor material.
- All circuit components are simultaneously formed by a process called planar process.
- There are various types of integrated circuits and for many different functions.

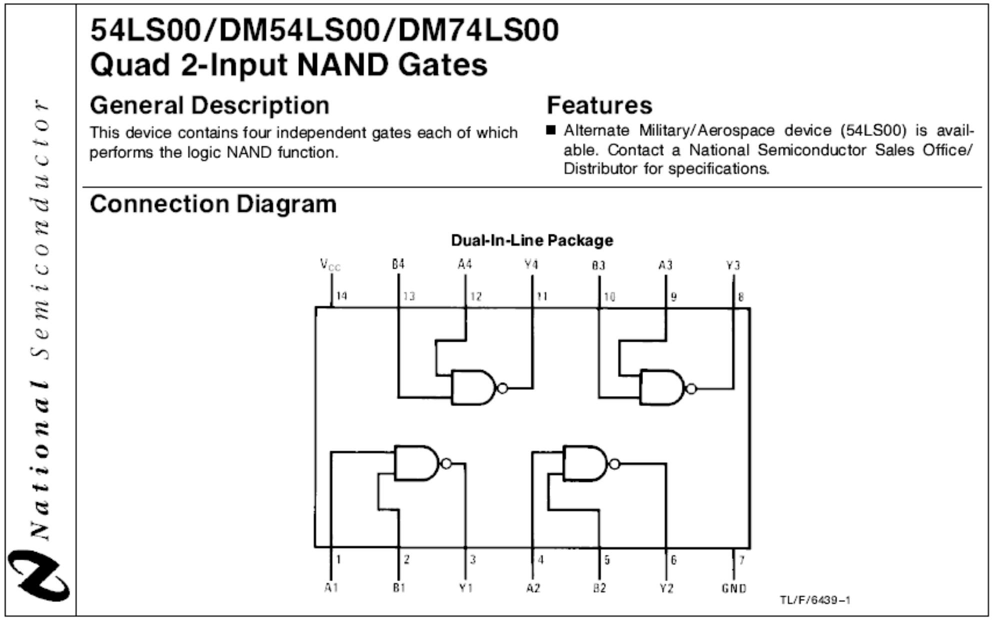
• A series of integrated circuits with the commonly used logic functions was created by a number of manufacturers (e.g. NAND gates, NOR, NOT, etc.)

• These devices are designed so that separate integrated circuits with different logical functions are electrically compatible with each other

• In practice it would not be possible to connect a silicon wafer directly to an electronic circuit.



• Identification of the pins of an integrated!


National Semiconductor

54LS00/DM54LS00/DM74LS00
Quad 2-Input NAND Gates

General Description

This device contains four independent gates each of which performs the logic NAND function.

Features

- Alternate Military/Aerospace device (54LS00) is available. Contact a National Semiconductor Sales Office/Distributor for specifications.

Connection Diagram

Dual-In-Line Package

TL/F/6439–1

• In a logical integrated circuit, the number of gates required to implement the functions depends on the complexity of the operation to be performed

Nível de Integração	Número de Portas
Integração em Pequena Escala (SSI)	<12
Integração em Média Escala (MSI)	[12,100[
Integração em Larga Escala (LSI)	[100,10000[
Integração em Muito Larga Escala (VLSI)	[10000,100000[
Integração em Ultra Larga Escala (ULSI)	≥100000

$V_{IH}(\min)$ Nível de tensão mínimo capaz de representar o nível lógico '1' à entrada de um circuito digital

$V_{IL}(\max)$ Nível de tensão máximo capaz de representar ainda o nível lógico '0' à entrada de um circuito digital

$V_{OH}(\min)$ Nível de tensão mínimo capaz de representar o nível lógico '1' à saída de um circuito digital

$V_{OL}(\max)$ Nível de tensão máximo capaz de representar o nível lógico '0' à saída de um circuito digital

I_{IH} Valor da corrente que circula na entrada de um circuito digital quando um nível lógico alto é aplicado.

I_{IL} Valor da corrente que circula na entrada de um circuito digital, quando um nível lógico baixo é aplicado.

I_{OH} Valor da corrente que circula na saída de um circuito digital, quando um nível lógico alto é gerado.

I_{OL} Valor da corrente que circula na saída de um circuito digital, quando um nível lógico baixo é gerado.

Recommended Operating Conditions

Symbol	Parameter	DM54LS00			DM74LS00			Units
		Min	Nom	Max	Min	Nom	Max	
V_{CC}	Supply Voltage	4.5	5	5.5	4.75	5	5.25	V
V_{IH}	High Level Input Voltage	2			2			V
V_{IL}	Low Level Input Voltage			0.7			0.8	V
I_{OH}	High Level Output Current			−0.4			−0.4	mA
I_{OL}	Low Level Output Current			4			8	mA
T_A	Free Air Operating Temperature	−55		125	0		70	°C

Electrical Characteristics over recommended operating free air temperature range (unless otherwise noted)

Symbol	Parameter	Conditions		Min	Typ (Note 1)	Max	Units
V_I	Input Clamp Voltage	V_{CC} = Min, I_I = −18 mA				−1.5	V
V_{OH}	High Level Output Voltage	V_{CC} = Min, I_{OH} = Max, V_{IL} = Max	DM54	2.5	3.4		V
			DM74	2.7	3.4		
V_{OL}	Low Level Output Voltage	V_{CC} = Min, I_{OL} = Max, V_{IH} = Min	DM54		0.25	0.4	V
			DM74		0.35	0.5	
		I_{OL} = 4 mA, V_{CC} = Min	DM74		0.25	0.4	
I_I	Input Current @ Max Input Voltage	V_{CC} = Max, V_I = 7V				0.1	mA
I_{IH}	High Level Input Current	V_{CC} = Max, V_I = 2.7V				20	μA
I_{IL}	Low Level Input Current	V_{CC} = Max, V_I = 0.4V				−0.36	mA
I_{OS}	Short Circuit Output Current	V_{CC} = Max (Note 2)	DM54	−20		−100	mA
			DM74	−20		−100	
I_{CCH}	Supply Current with Outputs High	V_{CC} = Max			0.8	1.6	mA
I_{CCL}	Supply Current with Outputs Low	V_{CC} = Max			2.4	4.4	mA

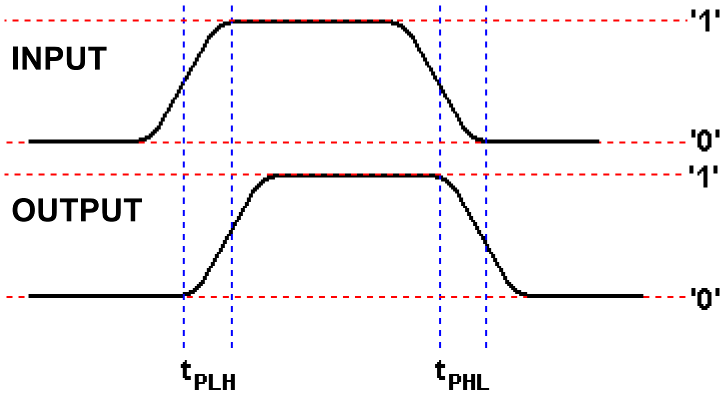
• Theoretically it is possible to connect an infinite number of logic gates to another logic gate

• *Fan-Out* is defined as the maximum number of logic gates that can be connected simultaneously to the output of another logic gate

$$FO_L = \frac{I_{OL}}{I_{IL}} \qquad FO_H = \frac{I_{OH}}{I_{IH}}$$

If FOL is different from FOH is considered the lesser of two!

Another feature to consider: <u>spread time of gates</u>

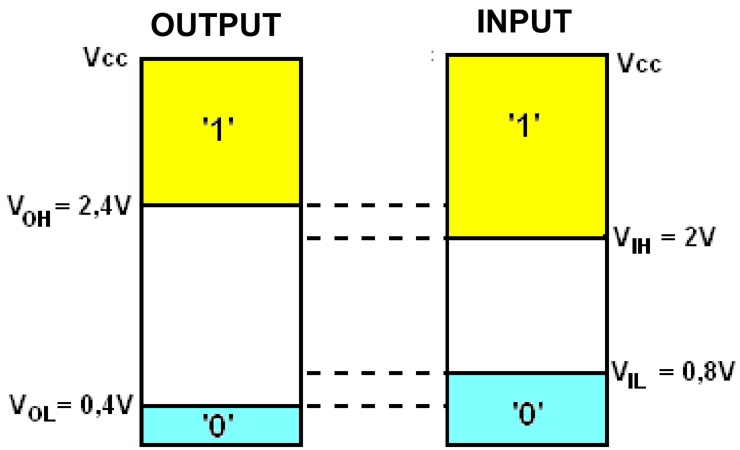


Noise immunity.

•The ability of a logic circuit tolerate voltage variations without changing its operation is quantified by the noise margin.

$$MR_H = V_{OH}(\min) - V_{IH}(\min)$$

$$MR_L = V_{IL}(\max) - V_{OL}(\max)$$



Logical IC families

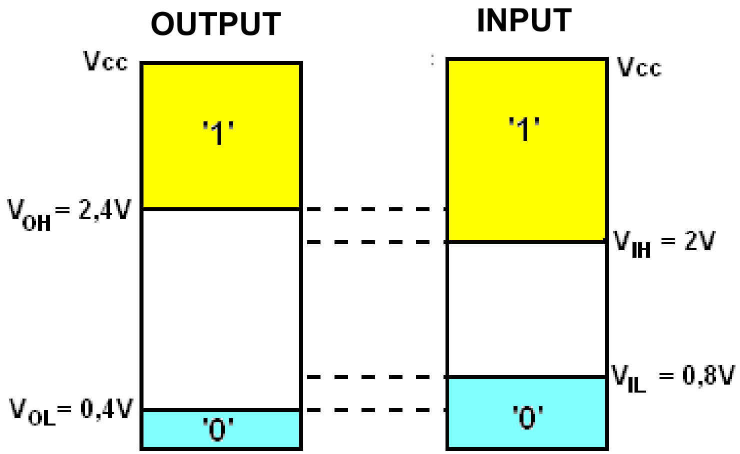
- TTL
- CMOS

Differences:

- Internal conception
- Voltage levels
- Other electrical characteristics

TTL

In the TTL family devices are powered by a direct voltage source of 5V



(deadband)

In TTL logic, a disconnected entry is considered as being to the high logic level.

- In order to define what kind of logical operation performs a TTL IC, these have a written reference on the casing.

Código	Significado	Particularidade
L	Low Power	Baixo consumo de potência quando comparada com a série padrão. Baixa velocidade de operação.(obsoleta)
H	High-Velocity	Maior velocidade de operação do que a série L mas maior consumo de potência.(obsoleta)
S	Schottky	Reduz o retardo de armazenamento aumentado a velocidade de operação. Consumo de potência equivalente à série H.
LS	Low-Power Schottky	Versão S com menor consumo e menor velocidade.
AS	Advanced Schottky	Série TTL mais rápida. Maiores Fan-Outs
ALS	Advanced Low-Power Schottky	Melhor desempenho que a série LS no que se refere à potência e velocidade de operação

Código	Designação
74LS00	4 Portas NAND de duas entradas
74LS02	4 Portas NOR de duas entradas
74LS04	6 Portas Inversoras
74LS08	4 portas AND de duas entradas
74LS10	3 portas NAND de três entradas
74LS11	3 portas AND de três entradas

CMOS

Main features

- Low power consumption
- High noise immunity
- Power range which may extend from 3 to 18V

The manufacturing process of CMOS technology is much simpler than the TTL further allowing a greater integration density

Disadvantages

- Lower operating speeds
- Homogeneity operating characteristics between manufacturers
- The range of values that represent the logical states are not constant

- Series 4000 and 14000 were the first from CMOS family.
- Latest series: 74C, 74HC and 74HCT.
- These last three sets have the particularity of being compatible pin-to-pin with its TTL counterparts

Código	**Designação**
4000	2 Portas NOR de três entradas e Inversor
4002	2 Portas NOR de quatro entradas
4012	2 Portas NAND de quatro entradas
74HC00	4 portas NAND de duas entradas
74HC107	Duplo Flip-Flop JK com Clear
74HC138	Descodificador 3 para 8

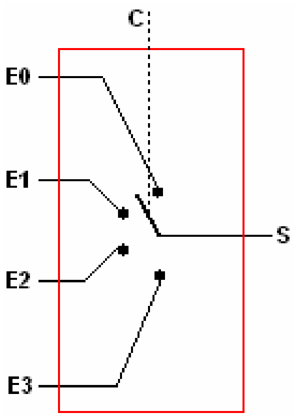
	74HC	**4000B**	**74**	**74S**	**74LS**	**74AS**	**74ALS**
Potência Dissipada (mW)	0,0025	0,001	10	20	2	8	1,2
Retardo Propagação (ns)	8	50	9	3	9,5	1,7	4
Produto velocidade/potência @100 KHz (pJ)	1,4	5	90	60	19	13,6	4,8
Máxima Frequência de Operação (MHz)	40	12	35	12,5	45	200	70
Margem de Ruído (V)	0,9	1,5	0,4	0,3	0,3	0,3	0,4

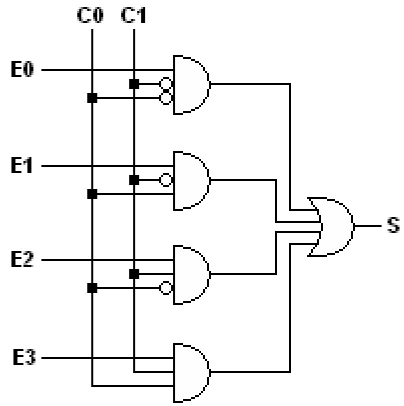
Multiplexers e Desmultiplexers

• The multiplex function consists on passing through one output channel some of the information present in several input lines.

• This type of device is constituted by a set of 2*n* inputs, only one output and a set of *n* control lines (address)

• For a given time instant, and depending on the state of the control lines, the output has the logic value identical to one and only one of its inputs.



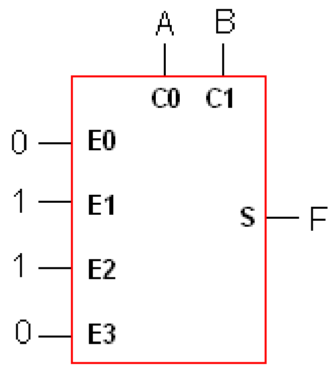


C0	C1	S
0	0	E0
0	1	E1
1	0	E2
1	1	E3

• As example of a commercial multiplexer in TTL technology one has, for example, the **74LS42**.

• A multiplexer can also be used to generate arbitrary logic functions of the control variables

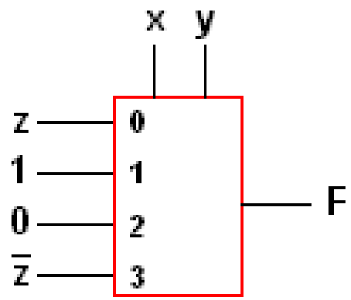
Example #1: EXCLUSIVE-OR function implementation



Example #2:

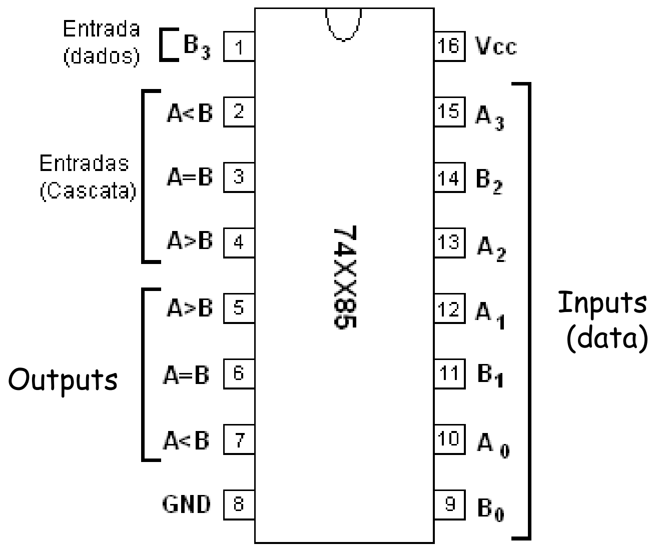
x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

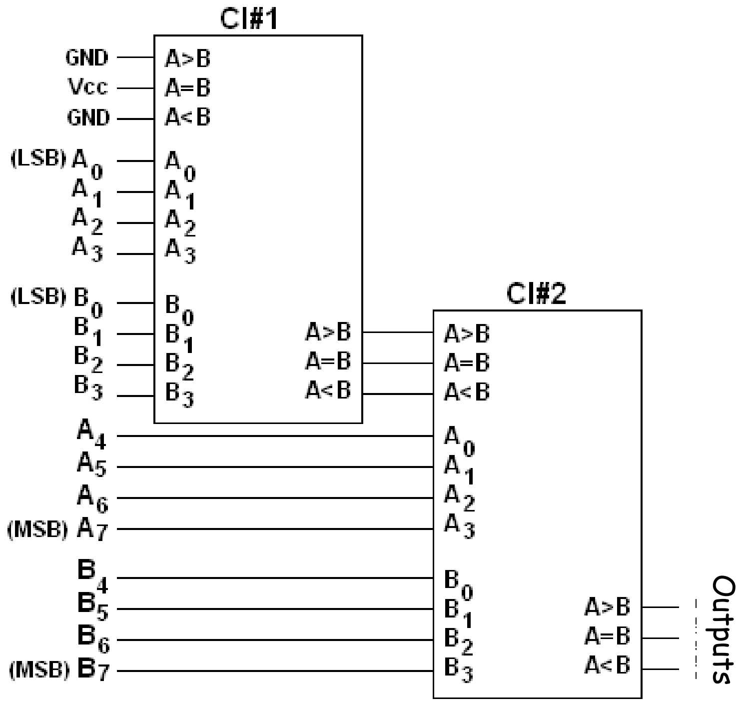
x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

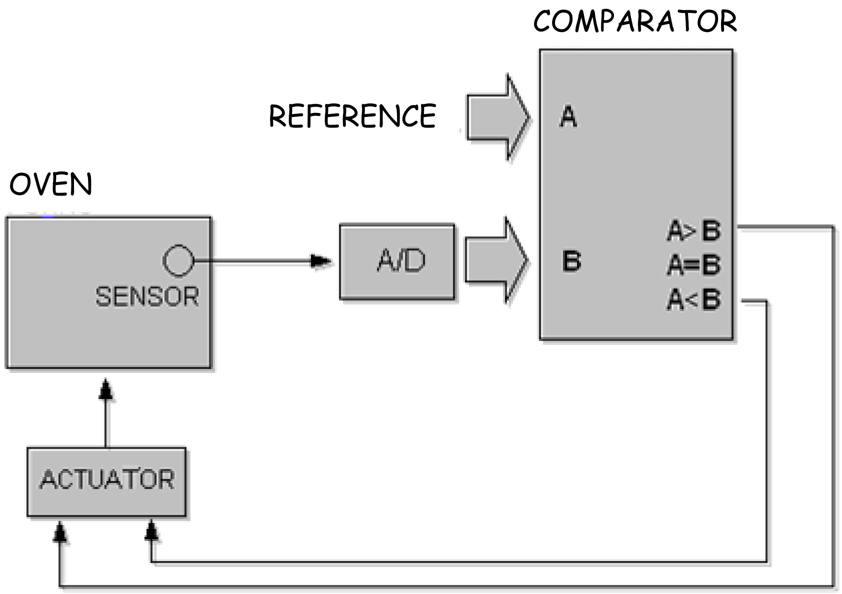


DIGITAL COMPARATORS

• The digital comparators are combinatorial circuits used to determine if two binary numbers are the same or different, and in this latter case, which of them is greater.






COMPARATOR
REFERENCE
A
OVEN
A/D
B
A> B
A=B
A<B
SENSOR
ACTUATOR