WORKING PROJECT 1

COMPUTATIONAL MECHANICS

Master of Science in

Construction Engineering
and
Industrial Engineering

INSTITUTO POLITÉCNICO DE BRAGANÇA
Escola Superior de Tecnologia e Gestão

1- Problem:

Calculate displacement on the tip of a rod, with varying cross section, axially loaded with force "F", as represented in figure 1. The student should test the convergence of the finite element analysis with mesh refinement, assuming three different meshes. Each finite element mesh should consider two possibilities (exact integration of the cross section and average cross section base on nodal cross section). Numerical solutions should be compared with analytical solution (exact).

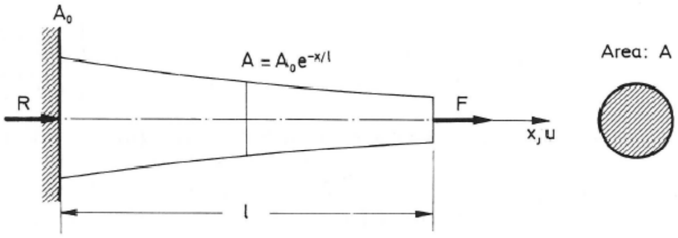


Figure 1 - Rod with variable cross section.

2- Analytical solution:

Assume displacement "u" defined over the coordinate x. Use first order approximation for normal strain in same direction, represented by equation (1).

$$\varepsilon_{xx} = du/dx \tag{1}$$

Introduce simplified Hook law, relating stress and strain, according to equation 2.

$$\sigma_{xx} = E\ \varepsilon_{xx} \tag{2}$$

Assume uniform normal stress distribution over the cross section area, being the longitudinal stress determined according to equation 3.

$$\sigma_{xx} = F/A \tag{3}$$

Displacement may be calculated with the following differential equation.

$$\frac{du}{dx} = \frac{F}{A_0 E} e^{x/L} \tag{4}$$

The general solution for displacement may be determined by exact integration.

$$u = \frac{FL}{A_0 E}\left(e^{x/L} - e^0\right) \tag{5}$$

The particular solution on the tip of the rod is calculated for x=L:

$$u = \frac{FL}{A_0 E}(1.71828) \tag{6}$$

3- Numerical solution:

The variational method applied to this element type, assuming linear displacement behaviour over the two node finite element may approximate the stiffness of this element. Equation 7 approximate the axial displacement as a function of two nodal parameters "ai".

$$u(x) = \langle 1 \quad x \rangle \begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix} \tag{7}$$

Those two nodal parameters should be determined according to the nodal displacement values, "ui". This approximation leads to the following linear interpolating functions.

$$u(x) = \langle N_i \quad N_j \rangle \begin{Bmatrix} u_i \\ u_j \end{Bmatrix} \tag{8}$$

$$N_i = 1 - \frac{x}{Le} \qquad N_j = \frac{x}{Le} \tag{9}$$

"Ni" and "Nj" represent the interpolating functions at node "i" and "j", respectively. see figure 2.

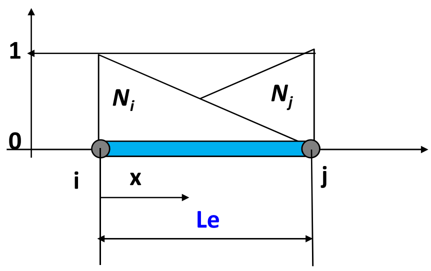
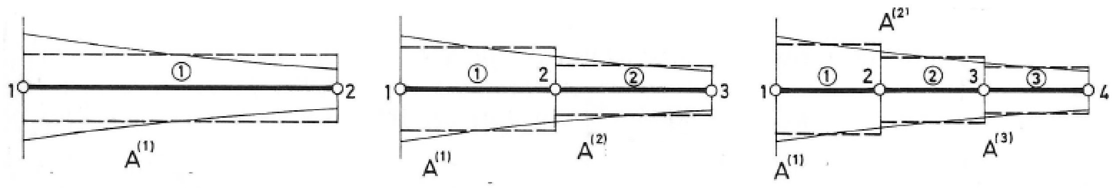


Figure 2 - Interpolating functions for finite bar element.

Applying the Minimum potential energy theorem, stiffness element matrix may be determined. This stiffness matrix depends on the element cross section, "A", between node "i" and "j". The area may be considered constant over the finite element length or may depends on. The elastic modulus "E" should be consider constant.

$$\left[K^e\right] = \int_0^{Le} EA^e \begin{bmatrix} \frac{dN_i}{dx}\frac{dN_i}{dx} & \frac{dN_i}{dx}\frac{dN_j}{dx} \\ \frac{dN_j}{dx}\frac{dN_i}{dx} & \frac{dN_j}{dx}\frac{dN_j}{dx} \end{bmatrix} dx \tag{10}$$

Student is invited to compare the accuracy of results with mesh refinement for the following numerical solutions, see figure 3.



a) b) c)

Figure 3 - Discrete approach for rod with variable section. a) mesh with one finite element. b) mesh with two finite element. c) mesh with three finite element.

The displacement field should be represented, in graphical excel format, over the length of the rod for each approximation. Displacement on the tip (x=L) should also be compared, determining the relative error, considering the numerical models suggested in table 1.

Table 1 - Numerical simulation for testing convergence.

Simulation	Nodal area	Number of elements
S1	Exact integration in stiffness	1
S2	Exact integration in stiffness	2
S3	Exact integration in stiffness	3
S4	$\overline{A}^{(1)} = \frac{1}{2} A_0 \left(1 + e^{-1}\right)$	1
S5	$\overline{A}^{(1)} = \frac{1}{2} A_0 \left(1 + e^{-1/2}\right)$ $\overline{A}^{(2)} = \frac{1}{2} A_0 \left(e^{-1/2} + e^{-1}\right)$	2
S6	$\overline{A}^{(1)} = \frac{1}{2} A_0 \left(1 + e^{-3}\right)$ $\overline{A}^{(2)} = \frac{1}{2} A_0 \left(e^{-1/3} + e^{-2/3}\right)$ $\overline{A}^{(3)} = \frac{1}{2} A_0 \left(e^{-2/3} + e^{-1}\right)$	3

4- Final statement:

Students are invited to present their work on standard report format (Word), until 2 weeks after receiving this working project.
Students are invited to present the main conclusions in next class, after delivering the report, using power point slides. The time schedule for each presentation is 15 minutes.