COMPUTATIONAL MECHANICS

Master of Science in

Construction Engineering
and
Industrial Engineering

INSTITUTO POLITÉCNICO DE BRAGANÇA Escola Superior de Tecnologia e Gestão

1- Problem:

Calculate buckling resistance for a squared hollow section steel profile S275, as represented in figure 1. The buckling resistance should consider different approaches:

      1- Linear elastic (Euler approach);
      2- Numerical linear elastic, using Eigen-value theory with ANSYS;
      3- Approximate results using Eurocode design buckling curves, [1];
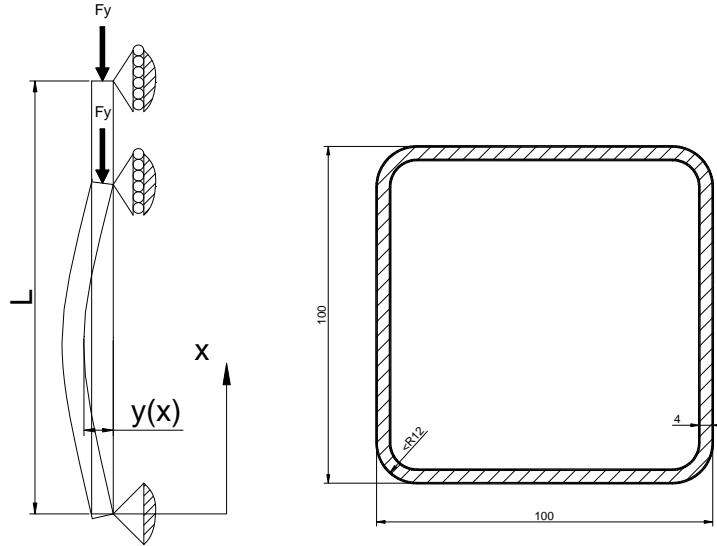   4- Geometry and material non - linear analysis, using ANSYS.

Figure 1 - Stability of SHS columns to compression and cross section dimensions.

For each solution method with different level of approximation, the student should calculate the buckling load, and normalize with respect to design resistance to normal forces of the cross-section, for uniform compression, (Mc,Rd), as represented in figure 2. The buckling resistance should be checked for different column lengths (L=1.21, 1.60, 2.00, 2.40, 2.80, 3.20, 3.40, 3.80, 4.20, 4.60, 5.00, 5.40, 6.00 and L=6.50 [m]).
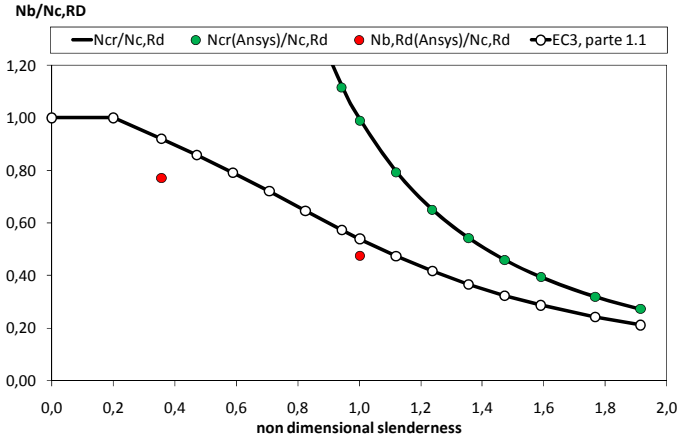

Figure 2 - Design buckling resistance for SHS, for different slenderness values.

The geometry parameters for this cross section are represented in table 1, and the dimensions represented in figure 1.

Table 1 - Geometry parameters.

Parameter	Value	Units
I_{zz}	2.34E-6	[m4]
A	1.53E-3	[m2]
Shear Z	12/5	[-]

The shear factor may de determined for different cross sections, using the following formula.

$$ShearZ = \frac{A}{I_{zz}^2} \int_A \frac{Q^2}{b^2} dA$$

(1)

Being A the cross section area, Izz the corresponding second order moment of area, Q represents the first order moment of area and b the width of the cross section.

2- - Linear elastic (Euler approach):

Assume second order differential equation to determine transversal displacement on beam column, represented by equation (1).

$$EI \frac{d^2 y}{dx^2} = M(x)$$

(2)

Assume the internal bending moment, produced by axial load offset, considering the equilibrium at the deformed configuration, according to equation 2.

$$M(x) = -Fy.y(x)$$

(3)

Using variable substitution according to equation 3.

$$P/EI = k^2$$

(4)

The second order differential equation becomes:

$$\frac{d^2 y}{dx^2} + k^2 y = 0$$

(5)

The general solution for displacement may be determined, considering the following solution for this ordinary differential equation.

$$y(x) = C_1 \sin(kx) + C_2 \cos(kx)$$

(6)

The Euler buckling load may be determined according to:

$$Fy_{cr} = \frac{n^2 \pi^2 EI}{L^2}$$

(7)

For the first shape mode, consider n=1.

3- Numerical linear elastic, using Eigen-value theory with ANSYS

For this numerical analysis, the method of eigen-value solution is required. Eigen-value buckling analysis predicts the theoretical buckling strength of an ideal elastic structure. It computes the structural eigen-values for the system of loading and constraints. This is known as classical Euler buckling analysis. Buckling loads for several configurations are readily available from tabulated solutions.

For this numerical approach, the student is invited to use finite BEAM 3 element, or equivalent. BEAM 3 is a linear elastic finite element, with three degrees of freedom (axial and transversal displacement and also nodal rotation).

The material behaviour is consider elastic, which means that the elastic modulus should be considered equal to 2.1 E11 [N/m2] and the Poisson coefficient equal to 0.3.

To perform an eigen-value buckling analysis, prestress effects must be activated. The eignen-value solver uses a unit force to determine the necessary buckling load. Applying a load other than 1 will scale the result by a factor dependent on load level. Apply a vertical (FY) point load of -1 N to the top of the column.

After applying the necessary boundary conditions, student must run a static analysis. However, with a buckling analysis student must re-enter the solution phase (after static analysis) and specify the second type of analysis (buckling analysis). Be sure to close the solution menu and re-enter it or the buckling analysis may not function properly.

4- Approximate results using Eurocode design buckling curves, [1];

Eurocode 3, [1], is required to design of steel structures or elements of structures. Any compression member should be checked against the Ultimate Limit State for stability. The inequality should be verified.

$$\frac{N_{Ed}}{N_{b,Rd}} \leq 1,0$$

(8)

N_{Ed} is the design value of the compression force and $N_{b,Rd}$ is the design buckling resistance of the compression member. This last value should be normalized against the design resistance to normal forces of the cross-section for uniform compression, Nc,Rd.

$$N_{c,Rd} = \frac{A f_y}{\gamma_{M,0}}$$

<div align="right">(9)</div>

The design buckling resistance of the compression member, Nb,Rd, may be calculated according to next equation.

$$N_{b,Rd} = \frac{\chi \, A \, fy}{\gamma_{M1}}$$

<div align="right">(10)</div>

χ is the reduction factor for the relevant buckling mode, and should be determined as function of the appropriate non-dimensional slenderness $\overline{\lambda}$.

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \overline{\lambda}^2}} \quad \wedge \quad \chi \leq 1$$

<div align="right">(11)</div>

The intermediate factor Φ should be determined according to:

$$\Phi = 0.5\left[1 + \alpha\left(\overline{\lambda} - 0.2\right) + \overline{\lambda}^2\right]$$

<div align="right">(12)</div>

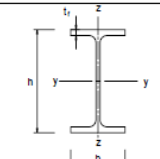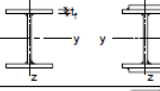Where the non dimensional slenderness for class 1,2 and 3 cross sections is calculated by:
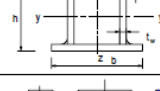
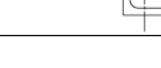$$\overline{\lambda} = \sqrt{\frac{A f_y}{N_{cr}}}$$

<div align="right">(13)</div>

The imperfection factor α corresponding to the appropriate buckling curve should be obtained from the next two tables.

Table 2 - Imperfection factor for each buckling curve.

Buckling curve	a_0	a	b	c	d
Imperfection factor α	0,13	0,21	0,34	0,49	0,76

Table 3 - Selection of the buckling curve for the each cross section.

5- Geometry and material non - linear analysis, using ANSYS

Nonlinear buckling analysis is more accurate than eigen-value analysis because it employs non-linear, large-deflection, static analysis to predict buckling loads. Its mode of operation is very simple: it gradually increases the applied load until a load level is found whereby the structure becomes unstable (ie. suddenly a very small increase in the load will cause very large deflections).

The true non-linear nature of this analysis thus permits the modeling of geometric imperfections and material nonlinearities. For this type of analysis, the student must define the imperfect geometry of the column. To define this geometric imperfection, it is acceptable to introduce a half sine harmonic, as represented by the next equation.

$$u = \frac{L}{1000}\sin\left(\frac{\pi x}{L}\right)$$

(14)

For this numerical approach, the student is invited to use finite BEAM 23 element, or equivalent. BEAM 23 is a non linear plastic finite element, with three degrees of freedom (axial and transversal displacement and also nodal rotation) that may be used for material and geometry non-linear buckling analysis.
The material model should be defined in to two distinct phases: Linear and non linear.
The linear material behaviour will be used for small strain, using the appropriate elastic modulus E, and the corresponding poisson coefficient.
The nonlinear material behaviour should be considered as: inelastic, rate independent, isotropic hardening plasticity, mises platicity, with multi-linear segment piecewise definition. The first data point should be used to define the first yielding of material, fy, as represented in the next figure. The first data point is also used to compare the elastic modulus. Ansys performs a comparison between the elastic modulus previously defined, E, and the elastic modulus determined using the first data point. The last one should be smaller or equal with respect to the previous defined E.
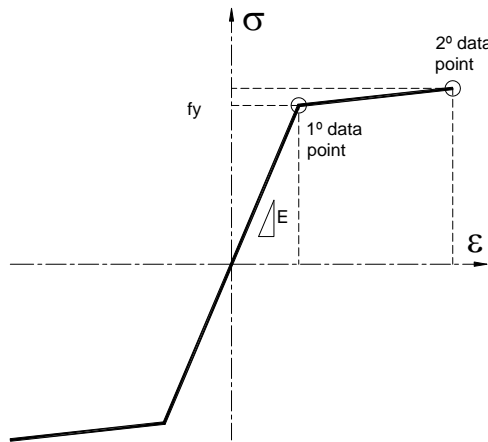

Figure 3 - Steel material behaviour, under tension and compression.

The load should be applied with "small" increments to update geometry, using a table defined with the Ansys utility menu. Apply for "Parameters", "array parameters", define or edit. Press "ADD" and change to "table" definition. Do not forget to introduce the name for the load parameter and to edit the values, according to the next table. This table defined parameter should be used when defining the loading boundary condition, Fy.

Table 3 - Load parameter to use as load variation.

Pseudo time	Parameter
0	-0.1
1000	-1000 000

Students must be advised that loading condition influences the numerical results, reason why particular careful should be cared.

4- Final statement:

Students are invited to present their work on standard report format (Word), until 2 weeks after receiving this working project.
Students are invited to present the main conclusions in next class, after delivering the report, using power point slides. The time schedule for each presentation is 15 minutes.

References:

[1] - CEN; EN 1993-1-1; Eurocode 3, Design of steel structures - Part 1-1: General rules and rules for buildings, May 2005.